Pathological tremors : Deterministic chaos or nonlinear stochastic oscillators?

Jens Timmer*, Siegfried Häußler*, Michael Lauk* and Carl Lücking†

* *Center for Data Analysis and Modeling, University of Freiburg, Eckerstr. 1, 79104 Freiburg, Germany*
† *Neurology Department, University Hospital of Freiburg, Breisacher Str. 64, 79110 Freiburg, Germany*

Abstract. Pathological tremors exhibit a nonlinear oscillation that is not strictly periodic. We investigate whether the deviation from periodicity is due to nonlinear deterministic chaotic dynamics or due to nonlinear stochastic dynamics. To do so, we apply methods from linear and nonlinear time series analysis to tremor time series. The results of the different methods suggest that the considered types of pathological tremors represent nonlinear stochastic second order processes.

INTRODUCTION

Tremor denotes an involuntary oscillation of parts of the body. Tremor time series span a large range of different dynamical behaviors. The physiological tremor of healthy subjects represents a linear second order stochastic process driven by white noise originating from uncorrelated firing motoneurons [1,2]. The enhanced physiological tremor can either be described by a stochastic linear second order process driven by colored noise or a nonlinear stochastic delay differential equation depending on the degree of the contribution of a central pacemaker or of reflexes [3,4]. Time series analysis yielded numerous insights in the underlying mechanisms of physiological as well as pathological forms of tremor in recent years [1–8].

Pathological tremors like essential and Parkinsonian tremor exhibit a nonlinear oscillation. The oscillation is not strictly periodic. There are at least three possible reasons for the deviation from a strictly periodic, limit cycle type of dynamics. First, deterministic chaos can be a source of apparently random behavior. Therefore, the processes have to be at least of third order. Second, the processes might be nonlinear stochastic oscillators, described by a second order differential equation with dynamical noise which is assumed to be additive. Third, the variability may result from nonstationarity in terms of time dependent parameters of the process.

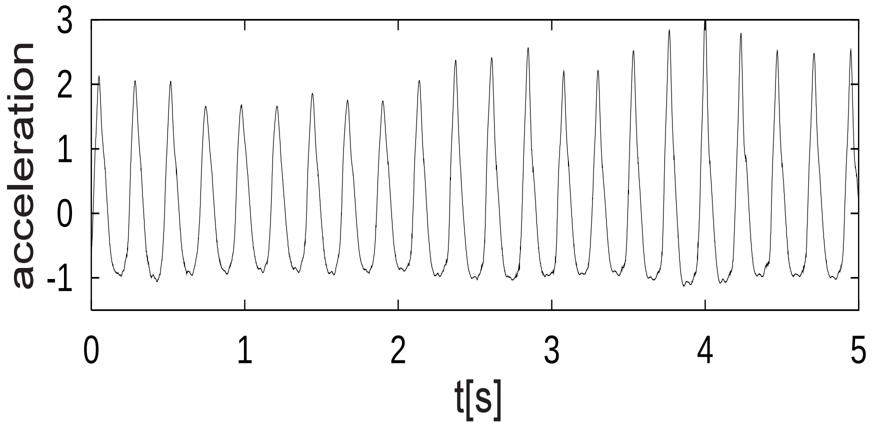

FIGURE 1. Segment of the time series of essential tremor.

Formally, such processes could be described by high-dimensional dynamical systems.

To evaluate whether a second order stochastic, a third order deterministic or a high-dimensional process underlies the measured time series, we investigated the local slopes of the correlation integrals, Poincaré and return maps, divergence of initially nearby trajectories, 'Deterministic versus stochastic'-plots and the $\delta - \epsilon$-method of time series from essential and Parkinsonian tremor time series [5]. In the following we present and discuss the results for the local slopes of the correlation integrals, the divergence of initially nearby trajectories and the 'Deterministic versus stochastic'-plots for a time series of essential tremor.

THE DATA AND THEIR SPECTRUM

The time series is a recordings of the acceleration of the hand measured by piezoresistive accelerometers attached to the dorsum of the out-stretched hand. The sampling rate is 1000 Hz. The data are recorded for 30 s yielding time series of length 30.000. For the analysis the time series were normalized to zero mean and unit variance. Fig. 1 shows a segments of the data for the essential tremor time series. Fig. 2 shows the estimated spectra based on the time series shown in Fig. 1. The higher harmonics in the spectrum indicate that the process underlying the time series is not a linear process, but some kind of nonlinear oscillator. The broadness of the peaks in Fig. 2 confirms the impression from Fig. 1 that the process is not a deterministic limit cycle.

NONLINEAR ANALYSIS

We aim to decide whether a nonlinear second order stochastic or a chaotic dynamic provide the most economical description consistent with the data. If this

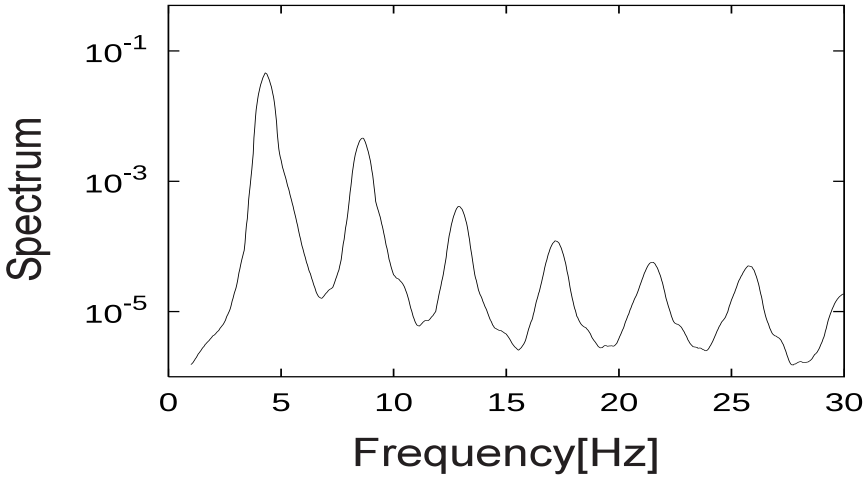

FIGURE 2. Estimated spectrum of the essential tremor.

question can not be decided in a conclusive way, some type of nonstationary, formally described as high-dimensional, process should be concluded. We investigated the applied algorithm with the Lorenz system and the stochastic van der Pol system:

$$\ddot{x} = \mu\,(1 - x^2)\,\dot{x} - x + \epsilon, \quad \mu = 3 \text{ and } \mu = 5\,.$$

The results of these simulation studies are given in [5].

Local slopes of correlation integrals

A fractal dimension of the attractor is one of the fascinating features of chaotic processes. One measure for this dimension is the correlation dimension D_2 [9]. It is defined by:

$$D_2 = \lim_{r \to 0} \frac{d\,\log C(r)}{d\,\log r} \quad . \tag{1}$$

$C(r)$, the correlation integral, is given by:

$$C(r) = \text{const} \sum_{i=1}^{N} \sum_{j=i+\mu}^{N} \Theta(r - |\vec{x}(i) - \vec{x}(j)|) \quad , \tag{2}$$

where $\vec{x}(i)$ denotes the states embedded in the reconstructed phase space, $\Theta(.)$ the Heavyside function applied to count the number of pairs of points within a radius r and μ the Theiler correction employed to exclude temporally correlated points.

A serious problem is that the existence of a scaling region for small radii r where Eq. (1) holds may not be assumed but has to be established. Therefore, plots of local slopes of the logarithm of the correlation integrals, Eq. (2), in dependence on the logarithm of r can be investigated. Evidence for a fractal attractor is given if the local slopes are constant for a large enough range of small radii and do not change for higher embedding dimensions. Fig.3 displays the result for the essential tremor time series. There is no indication for a low dimensional fractal attractor.

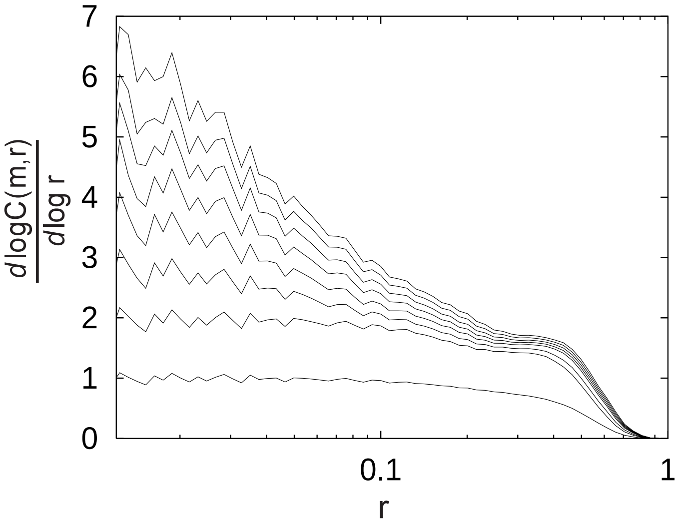

FIGURE 3. Local slopes of the correlation integrals for the essential tremor time series.

Divergence of initially nearby trajectories

A positive Lyapunov exponent is one of the hallmarks of chaos. The local exponential divergence of nearby trajectories in chaotic systems lead to the sensitivity of the trajectories against small changes in the initial values. However, also in stochastic dynamical systems nearby trajectories diverge. But their divergence does not follow an exponential law. Therefore, it is important to establish the exponential divergence to find an indicator for a chaotic dynamics. In the following we use the algorithm suggested in [10]. For initial neighborhoods $\mathcal{U}_i, i = 1, \ldots, N$ of size ϵ in a m dimensional reconstructed phase space the quantity

$$S(\epsilon, m, t) = \left\langle \ln \left(\frac{1}{|\mathcal{U}_i|} \sum_{\vec{x}(j) \in \mathcal{U}_i} |\vec{x}(i+t) - \vec{x}(j+t)| \right) \right\rangle_i \tag{3}$$

is evaluated in dependence on t. A plot of $S(\epsilon, m, t)$ versus t displays a straight line in the case of an exponential divergence of initially nearby trajectories in a chaotic system. Fig.4 displays the result for the essential tremor time series. There is a divergence of initially nearby trajectories but it does not follow an exponential behavior. In fact, we observed similar curves for the nonlinear stochastic van der Pol oscillator [5]. The invariance of the resulting curve for embedding dimension $m \geq 2$ indicates a second order dynamics.

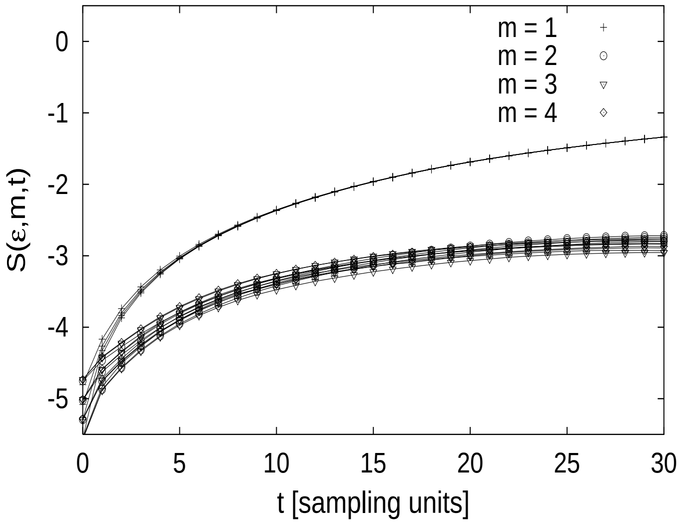

FIGURE 4. Local divergence of initially nearby trajectories for the essential tremor time series for embedding dimension m=1,..,4.

Deterministic versus Stochastic plots

'Deterministic versus stochastic' (DVS) -plots were introduced in [11]. For different embedding dimensions, this method applies local linear predictions of the time series using different large neighborhoods for the local linear modeling and evaluates the mean prediction error. The information provided by this method is twofold. First, for sufficiently high embedding dimensions, nonlinear deterministic, nonlinear stochastic and linear stochastic processes result in different appearances of the DVS plots. For nonlinear deterministic processes the prediction error approaches zero for small neighborhoods and increases monotonically. For linear stochastic processes, the prediction errors should be largest for the smallest neighborhoods and decrease monotonically. Depending on the amount of dynamical noise and the degree of nonlinearity, nonlinear stochastic processes show either a minimum error in an intermediate size of neighborhoods or a monotonically increasing prediction error that does not reach zero for small neighborhoods.

Second, the order of the process can be inferred. For embedding dimensions larger than the true order, the prediction errors should not decrease any further and the functional form of the prediction errors in dependence on the size of the neighborhoods should not change anymore.

Fig. 5 displays the results for the tremor data. A second order process is suggested by the invariance of the functional form of the prediction error curves for higher embedding dimensions. The nonlinearity of the processes is reflected by the increase of the prediction error for larger ϵ. For small ϵ the prediction error shows a tendency to increase. Thus, the DVS-plot give evidence for a stochastic nonlinear second order dynamics.

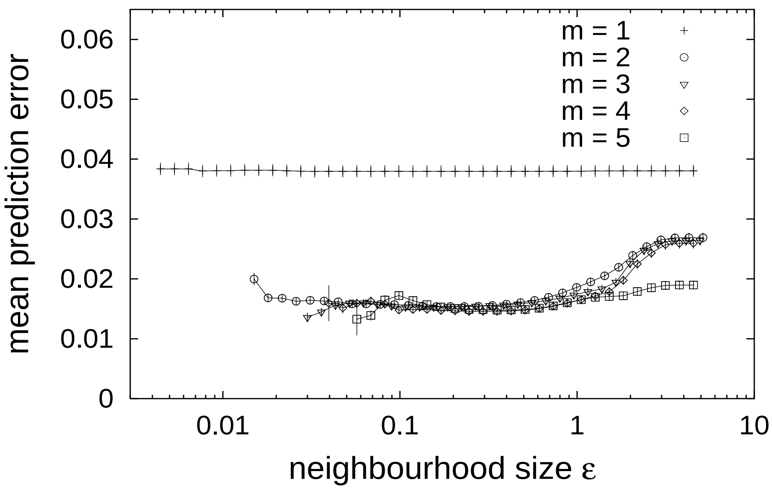

FIGURE 5. DVS-plots for embedding dimension one to five.

CONCLUSION

The results suggest that the investigated time series of essential tremor is consistent with a second order nonlinear stochastic dynamics.

The time series used in the above investigations and further essential and Parkinsonian tremor time are available at:
http://phym1.physik.uni-freiburg.de/~jeti/path_tremor

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